

DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Transfer all your answers to the space provided.

1. The given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

$$y = c_1 e^{4x} + c_2 e^{-x}, \quad (-\infty, \infty); \quad y'' - 3y' - 4y = 0, \quad y(0) = 1, y'(0) = 1$$

2. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

a. $f_1(x) = 0, \quad f_2(x) = x, \quad f_3(x) = e^x$

b. $f_1(x) = 3 + x, \quad f_2(x) = x, \quad f_3(x) = x^2$

3. Consider the differential equation $y'' - y' - 12y = 0$
Use the Wronskian to Verify that the functions $y_1 = e^{-3x}, y_2 = e^{4x}$ form a fundamental set of solutions of the differential equation on the interval $(-\infty, \infty)$, then give the general solution.

4. Consider the differential equation
 $x^2 y'' + xy' + y = 0; \cos(\ln(x)), \sin(\ln(x)), (0, \infty)$.
Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval, then form the general solution.