Math002A Assignment 5.1 Last Name:____________________1st_________

DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided.

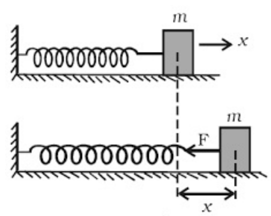
Mechanical Vibration: Free Undamped Motion

1. A body with mass 250 g is attached to the end of a spring that is stretched to 25 cm by a force of 15 N. At time t = 0 the body is pulled 1 m to the right, stretching the spring, and set in motion with an initial velocity of 5 m/s to the left.
The only force acting on the spring is the sprig restoring force and the governing law of motion is Newton's second Law

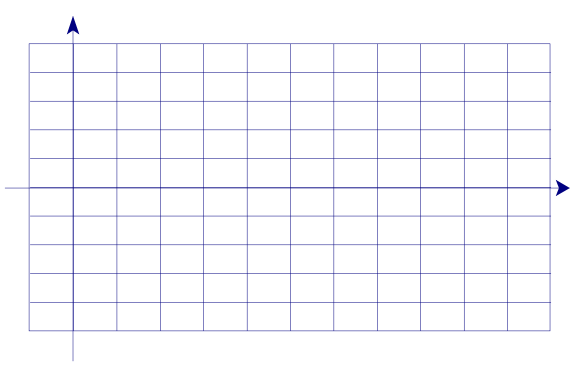
a. Find the IVP that represents the motion of the mass a function of time, for convenience, write $\omega_0 = \sqrt{k/m}$

b Find the position of the body as a function of time in the form $x(t) = C\cos(\omega_0 t - \alpha)$

c. Find the amplitude, the frequency and the period of motion of the body and sketch its position graph.



Linear harmonic oscillator



Mechanical Vibration: Free Damped Motion

Problems 2, 3 and 4.

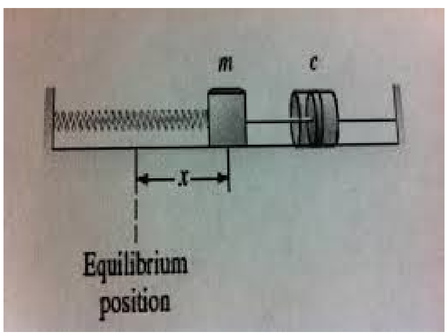
A mass M g is attached to both a spring, with spring constant k, and a dashpot, with damping constant c. Themass is set in motion with initial position x_0 and an initial velocity v_0. For convenient, write $\omega_0 = \sqrt{k/m}$

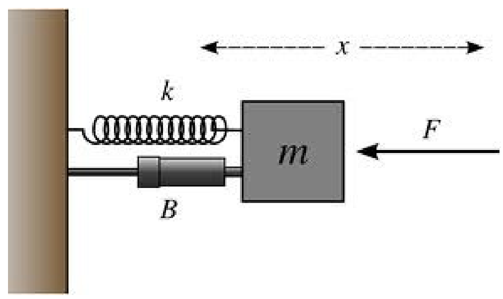
a. Find the IVP that represents the motion of the mass a function of time.

b. Find the position of the mass as a function of time and determine wether the motion is over damped, critically damped or under damped. (Definition in chapter 5).

c. If the motion is under damped, write the position function in the form

d. And find the free undamped position $x(t) = C\cos(\omega_0 t - \alpha)$ that would result if the mass on the spring were with the same initial position and velocity, but with the dashpot disconnected, then graphically compare the effect of damping by graphing both x and u on the same axes.





Assumptions:

The force **F**r provided by the dashpot is proportional to the velocity of mass

$\text{Fr} = cx'$ Where c is the dashpot's damping constant.

The force **F**x, the restoring force of the sprig is proportional to the distance of the mass from the equilibrium position.

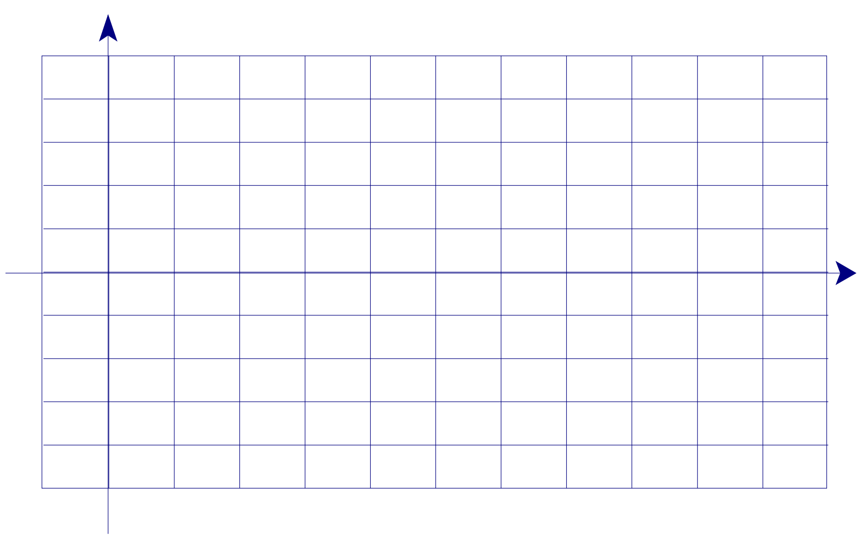
$\text{F}x = kx$, where k is the spring constant.

External force when present **F**e

Total force acting on the mass: **Fx + Fr + Fe**

The governing law of the motion, Newton's second Law: $F = ma$

2. $m = 3kg,\ c = 30N \cdot S / m,\ k = 63N / m,\ x_0 = 2m,\ x_0 = 2m / s$



3. $m = 1kg,\ c = 8N \cdot S / m,\ k = 16N / m,\ x_0 = 5m,\ x_0 = -10m / s$