

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Find the distance between \mathbf{u} and \mathbf{v} , $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

2. Let W be a subspace of \mathfrak{R}^n and W^\perp is the orthogonal complement of W . Prove W^\perp is a vector subspace of \mathfrak{R}^n . Hint Show that W^\perp is closed under vector addition and scalar multiplication.

3. Let $W = \text{span}\{u_1, u_2, \dots, u_m\}$

Show that if \mathbf{x} is orthogonal to every vector u_i , $1 \leq i \leq m$, then it is contained in the orthogonal complement of W . Hint let u be an arbitrary vector in W

4. Determine which of the given sets is orthogonal?

a. $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ b. $\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

5. Show that $\{u_1, u_2, u_3\}$ is an orthogonal basis for set of real numbers \mathbb{R}^3 .
Then express \mathbf{x} as a linear combination of the \mathbf{u} 's.

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}.$$

6. Express the given vector \mathbf{v} as a linear combination of vectors \mathbf{u} and another vector orthogonal to \mathbf{u} .

$$\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}, \quad \mathbf{u} = \mathbf{i} + \mathbf{j}$$