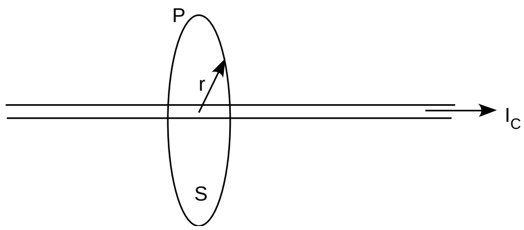
MAXWELL'S DISPLACEMENT CURRENT

A. Recall Ampere's Law



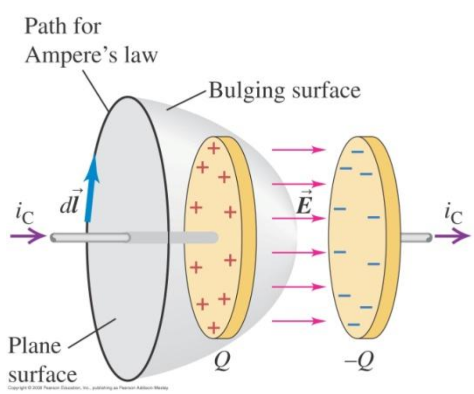
$$\oint \vec{B} \bullet d\vec{l} = \mu_o I_c$$

The line integral of $\vec{B} \bullet d\vec{l}$ around **any closed path** equals $\mu_o I_C$ where I_C is the total steady-state conduction current passing through **any surface** bounded by the closed path.

$$B 2\pi r = \mu_o I_c$$

$$B = \frac{\mu_o I_c}{2\pi r}$$

B. Maxwell's Difficulty
Consider a charging capacitor.



Consider the two surface – the plane surface and the bulging surface:

Plane Surface

$$\oint_{\substack{plane\\surface}} \vec{B} \bullet d\vec{l} = \mu_o I_c$$

Bulging Surface

$$\oint_{\substack{bulging\\surface}} \vec{B} \bullet d\vec{l} = 0 \quad \text{Thus, clearly a contradiction!!}$$

C. Maxwell's Solution

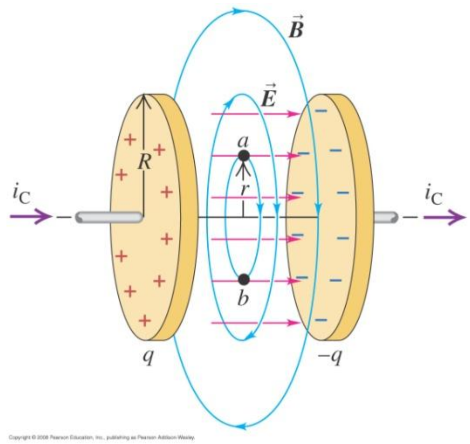
Maxwell solved this problem by postulating an additional term in Ampere's Law called Displacement Current.

$$I_d = \varepsilon_o \frac{d\Phi_E}{dt}$$ Displacement Current

$$\Phi_E = \int \vec{E} \bullet d\vec{A}$$

Where did this "Displacement Current" come from????

For the 'bulging surface' we can see that the E-field and thus the flux Φ_E are changing as capacitor is being charged. To find the rate at which they change (increase):



$$E = \frac{\sigma}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$$

$$q = \varepsilon_o EA$$

$$I_c = \frac{dq}{dt} = \varepsilon_o \frac{d(EA)}{dt} = \varepsilon_o \frac{d\Phi_E}{dt} = I_d$$

Thus,

$$I_c = I_d$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_o (I_c + I_d) = \mu_o I_c + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$ Maxwell-Ampere's Law

Magnetic fields are produced both by conduction currents and by a changing electric flux.